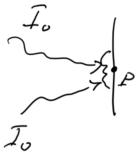
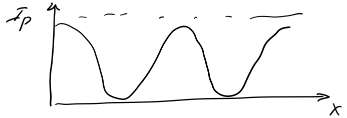
Multi-beam interference

So far we only considered the case of two interfering beams.



$I_1 = I_2 = I_0$

$I_p = 2I_0(1 + \cos k\Delta)$

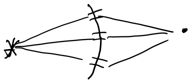


What happens if many beams will come to the point P?

$I_p = ?$

Multi-beam interference can be realized again by two methods:

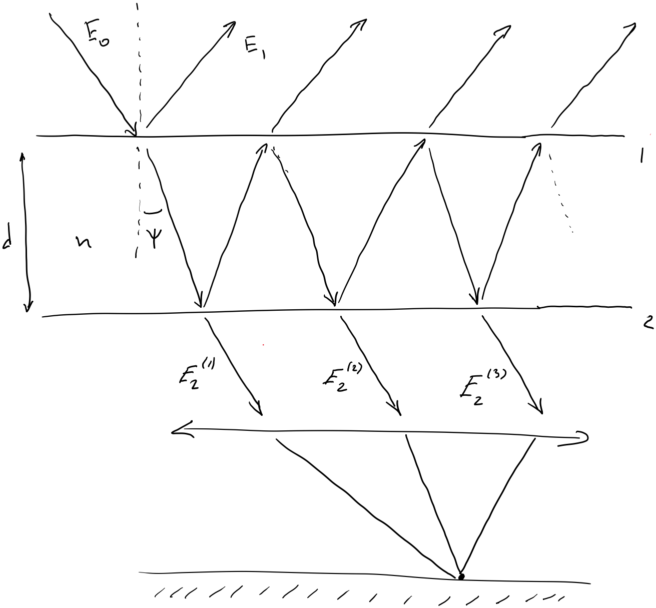
1) Method of wave front division (diffraction)



2) Method of amplitude division

Fabry-Perot interferometer

$R \to 1$



Compared to interference model in thin film, here the surface reflection is $R \to 1$.

$E_2^{(1)}$, $E_2^{(2)}$ and $E_2^{(3)}$ are small as $R \to 1$.

$E_2^{(1)}$, $E_2^{(2)}$ and $E_2^{(3)}$ are coherent

① We need to show, that $E_2^{(1)}$, $E_2^{(2)}$ and $E_2^{(3)}$ are equal in intensity.

② We need to demonstrate, that optical path difference

$$\Delta\left(E_2^{(i+1)} - E_2^{(i)}\right) = 2dn\cos\Psi$$

$$\delta = k_0 \Delta = \frac{2\pi}{\lambda_0} 2 d n \cos\psi$$

phase difference

Let's introduce amplitude reflection coefficient

$$\rho = \frac{E_1}{E_0}$$

⇝ reflected amplitude

↪ incident amplitude

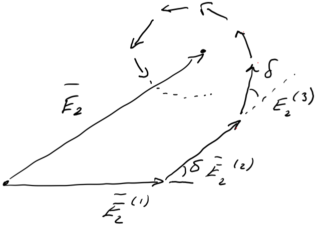
Amplitude transmission coefficient

$$\tau = \frac{E_2}{E_0}$$

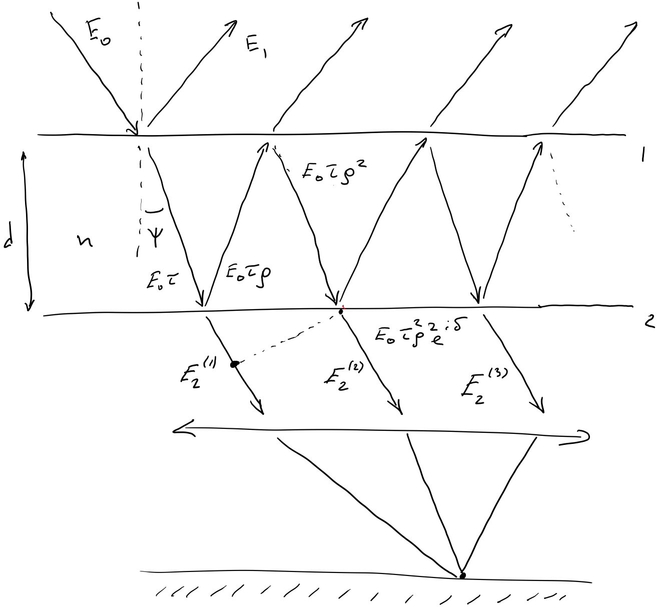
Transmitted field can be written:

$$E_2 = \sum_{i=1}^{\infty} E_2^{(i)}$$

We can use vector diagram:



Let's sum it up



$$\left.\begin{aligned}
E_2^{(1)} &= E_0\tau \\
E_2^{(2)} &= E_0\tau^2\rho^2e^{i\delta} \\
E_2^{(3)} &= E_0\tau^2\rho^4e^{i2\delta} \\
&\cdots\cdots \\
E_2^{(n)} &= E_0\tau^2\rho^{2(n-1)}e^{i(n-1)\delta}
\end{aligned}\right\} +$$

$$E_2 = E_0\tau^2(1+\rho^2e^{i\delta}+\rho^4e^{i2\delta}+\ldots+\rho^{2(n-1)}e^{i(n-1)\delta}+\ldots)$$

Σ is infinitly decreasing geometrical progression.

$$\Sigma = \frac{1}{1-q} \qquad q = \rho^2 e^{i\delta}$$

$$E_2 = E_0 \tau^2 \frac{1}{1-\rho^2 e^{i\delta}}$$

Let's move now to intensities

$$R = \rho^2 = \frac{I_1}{I_0}$$

$$T = \tau^2 = \frac{I_2}{I_0}$$

$$R + T = 1 \longrightarrow \text{no losses}$$

$$E_2 = E_0 \frac{T}{1 - Re^{i\delta}}$$

$$\frac{I_2}{I_0} = \frac{E_2 E_2^*}{I_0} = \frac{T^2}{(1-Re^{i\delta})(1-Re^{-i\delta})}$$

Let's simplify it!

$$(1-Re^{i\delta})(1-Re^{-i\delta}) = 1 + R^2 - 2R\cos\delta =$$

$$= 1 + R^2 - 2R\left(1 - 2\sin^2\frac{\delta}{2}\right) = (1-R)^2 +$$

$$+ 4R\sin^2\frac{\delta}{2}$$

$$\frac{I_2}{I_0} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

Let's introduce coefficient F:

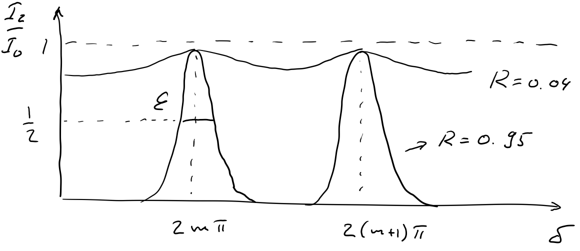
$$F = \frac{4R}{(1-R)^2}$$

$$\boxed{\frac{I_2}{I_0} = \frac{1}{1 + F \sin^2 \frac{\delta}{2}}}$$ Airy equation

$$F = \begin{cases} \approx 0.2 & R = 0.04 \quad \text{air - glass} \\ \approx 1600 & R = 0.95 \quad \text{mirror} \end{cases}$$

$$\delta = k_0 \Delta = \frac{2\pi}{\lambda_0} 2 d n \cos\psi$$

If we are going to change incident angle φ, i.e. ψ, then we will have interference pattern.

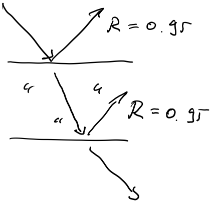


$$\left(\frac{I_2}{I_0}\right)_{max} \Rightarrow \delta = 2m\pi$$

It means that Airy function can reach 1

$$\frac{I_2}{I_0} = 1$$

How can we understand it if $R = 0.95$?



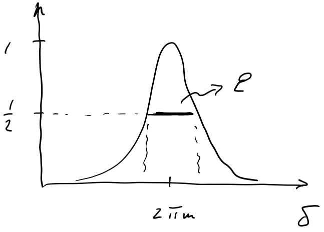
How?

$$\left(\frac{I_2}{I_0}\right)_{min} = \frac{1}{1+F} = \begin{cases} \frac{1}{1.2} \approx 0.8 & \text{when } R = 0.04 \\ \frac{1}{1600} & \text{when } R = 0.95 \end{cases}$$

That is why we cannot observe interference on thin film in transmission geometry

What is the width of our interference strips?

Let's find width ε



$$\frac{1}{2} = \frac{1}{1 + F \sin^2\left(\frac{2\pi m + \frac{\varepsilon}{2}}{2}\right)} = \frac{1}{1 + F\sin^2\frac{\varepsilon}{4}} \approx$$

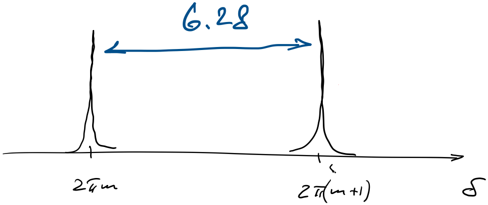
1

$$\approx \frac{\qquad}{1+F\left(\frac{\varepsilon}{4}\right)^{2}}$$

$$\varepsilon = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}$$

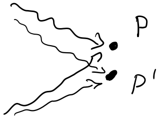
Hence, when $R = 0.95$ $\varepsilon = 0.1$.

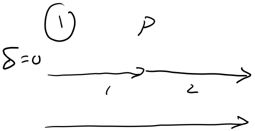
very narrow!

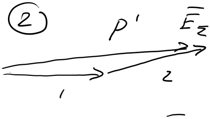


Why these interference lines are this narrow?

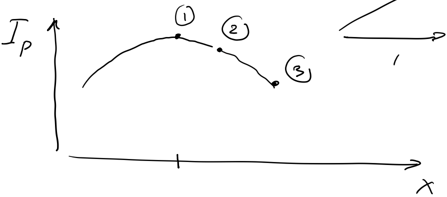
Let's first consider two beam interference.

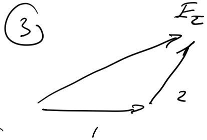






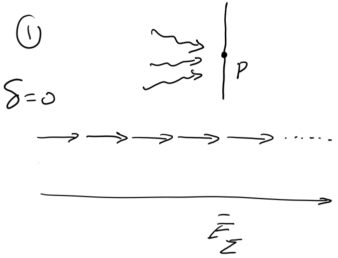
$\bar{E}_\Sigma$





In case of multi-beam interference the problem is more sophisticated.

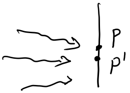
①



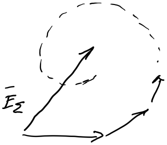
Estimate how many beams are interfering when $R = 0.95$

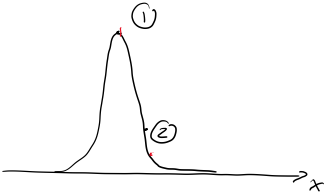
$E_2^{(n)} = 0.2E_2^{(1)}$

②



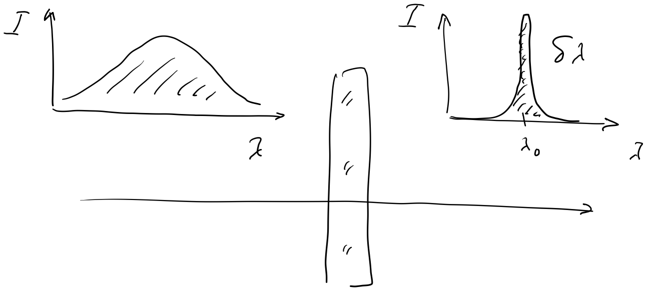
Each vector will be shifted by δ in phase





Interference filter

We can use this principle to transmit or reflect very narrow spectral band.



Demonstration

Let's explain this.

$$\delta = \frac{2\pi}{\lambda_0} 2dn\cos\varphi \qquad \varphi = 0$$

$$\delta = \frac{2\pi}{\lambda_0} 2dn$$

Condition for having maximum

$$\delta = 2\pi m$$

That will correspond to

$$\lambda_m = \frac{2dn}{m}$$

If we take $\lambda = 600$ nm and $m = 3$

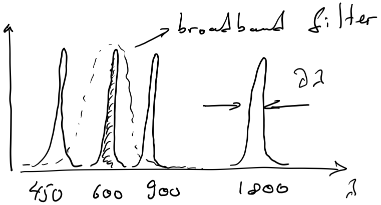
$$2dn = 600 \cdot 3 = 1800 \text{ nm}$$

$$d = \frac{1800 \text{ nm}}{2n}$$

However, this filter will transmit other wavelength, that

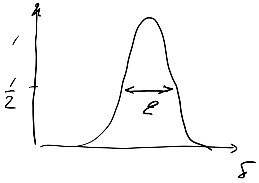
$$\underline{2dn = m\lambda} \Rightarrow$$

1800 nm $(m=1)$
900 nm $(m=2)$
450 nm $(m=4)$



What spectral width our filter will have?

$$\partial\lambda = ?$$



$$\delta = \frac{2\pi}{\lambda_0} 2dn$$

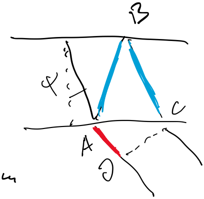
$$\varepsilon \equiv d\delta = \frac{2\pi}{\lambda_0^2} 2dn\, \partial\lambda$$

$$\partial\lambda = \frac{\lambda_0^2\,\varepsilon}{2\pi\,\underbrace{2dn}_{m\lambda_0}} = \frac{\lambda_0\,\varepsilon}{2\pi m} = \frac{\varepsilon}{2\pi}\cdot\frac{\lambda_0}{m}$$

For our filter

$$\partial\lambda = \frac{\varepsilon}{2\pi}\cdot 200\text{ nm} \qquad \varepsilon = 0.1\ (R = 0.95)$$

$$\partial\lambda = 3\text{ nm}$$



$$\delta = \frac{2\pi}{\lambda_0}\,2dn\cos\varphi = 2\pi m$$

Demonstration of Fabry-Perot interferometer

1) Na lamp

$\left\{\begin{array}{l}589\text{ nm}\\ 589.6\text{ nm}\end{array}\right.$ Which line is closer to the center?

2) If we start to pump the air out, why do rings go closer to the center?